Recall

- GLM is flexible
  - One Sample T Test
  - ANOVA
  - Two sample T Test
  - Paired T test
- What do the models look like?

1-Sample T Test

$$X\beta = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \beta$$

← Overall mean

$$H_0 : c\beta = 0 \quad \text{where} \quad c = [1]$$

2-Sample T Test

$$\begin{pmatrix} A_1 \\ A_1 \\ A_1 \\ A_1 \\ A_1 \\ A_2 \\ A_2 \\ A_2 \\ A_2 \\ A_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix}$$

Mean group 1 (β_1)

Mean group 2 (β_2)

$$H_0 : c\beta = 0 \quad \text{where} \quad c = [1 \quad -1]$$

2-Sample T Test

OR

$$\begin{pmatrix} A_1 \\ A_1 \\ A_1 \\ A_1 \\ A_1 \\ A_2 \\ A_2 \\ A_2 \\ A_2 \\ A_2 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix}$$

Understanding a model

- If you're unsure about a model or the contrasts
  - Plug in numbers
  - Look at graphs (fMRI data)
- Always ask yourself if your model is doing what you want it to

For example…

- For the 2 sample T test
  - Set $\beta_1 = 3 \quad \beta_2 = 5$
  - Then G1=8 and G2=3
  - So β_1 is the mean of group 2 and β_2 is the difference between the groups
  - What are the contrasts to test
    - Mean of G2 $c = [1 \quad 0]$
    - Mean of G1
    - G1-G2

$$X\beta = \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix}$$

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    - Mean of G1 $c = [1 \quad 1]$
    - G1-G2

$$X\beta = \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix}$$

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    - Mean of G2 $c = [1 \quad 0]$
    - Mean of G1 $c = [1 \quad 1]$
    - G1-G2 $c = [0 \quad 1]$

$$X\beta = \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix}$$

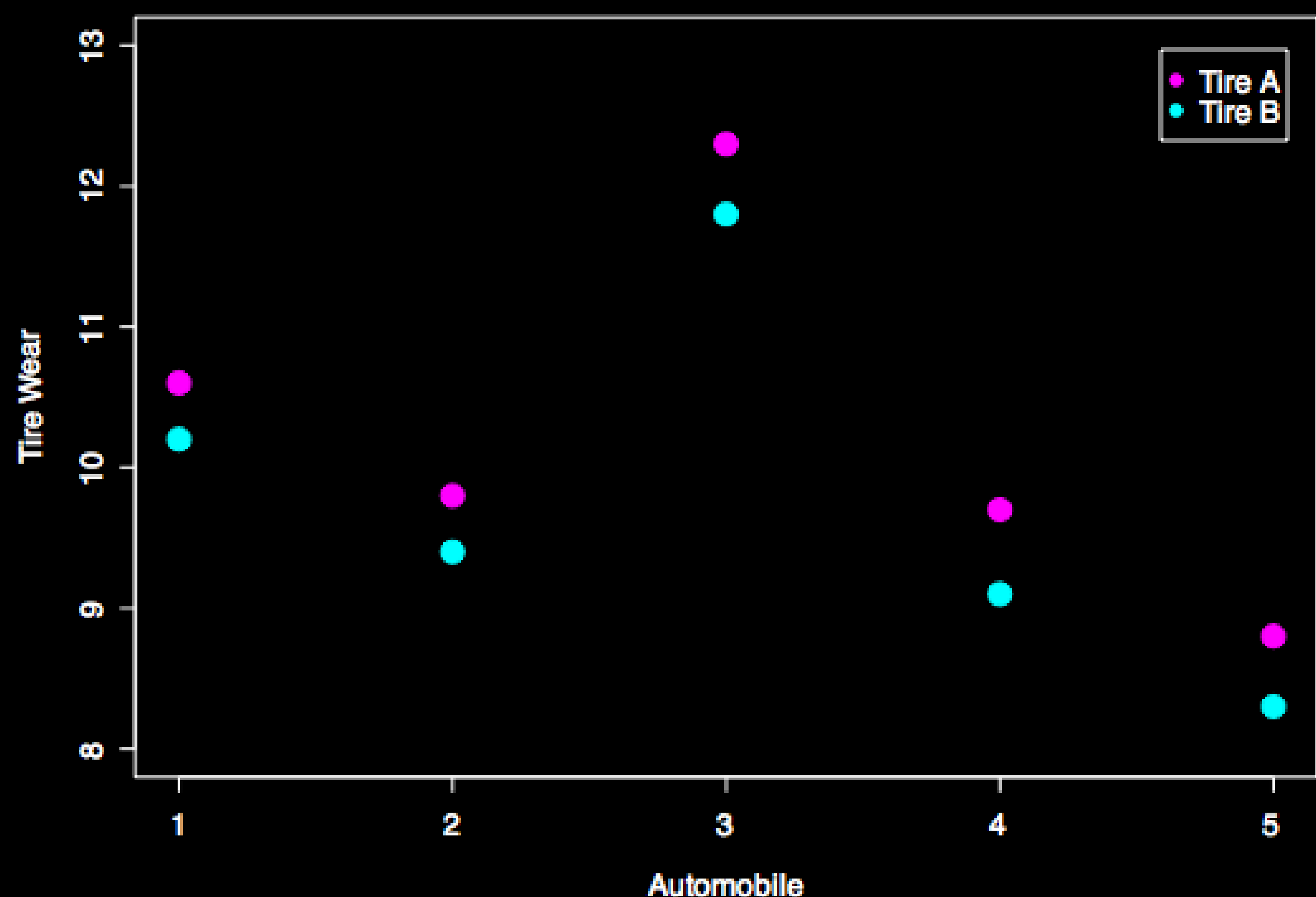
Paired T Test

- A common mistake is to use a 2-sample t test instead of a paired test
- Tire example

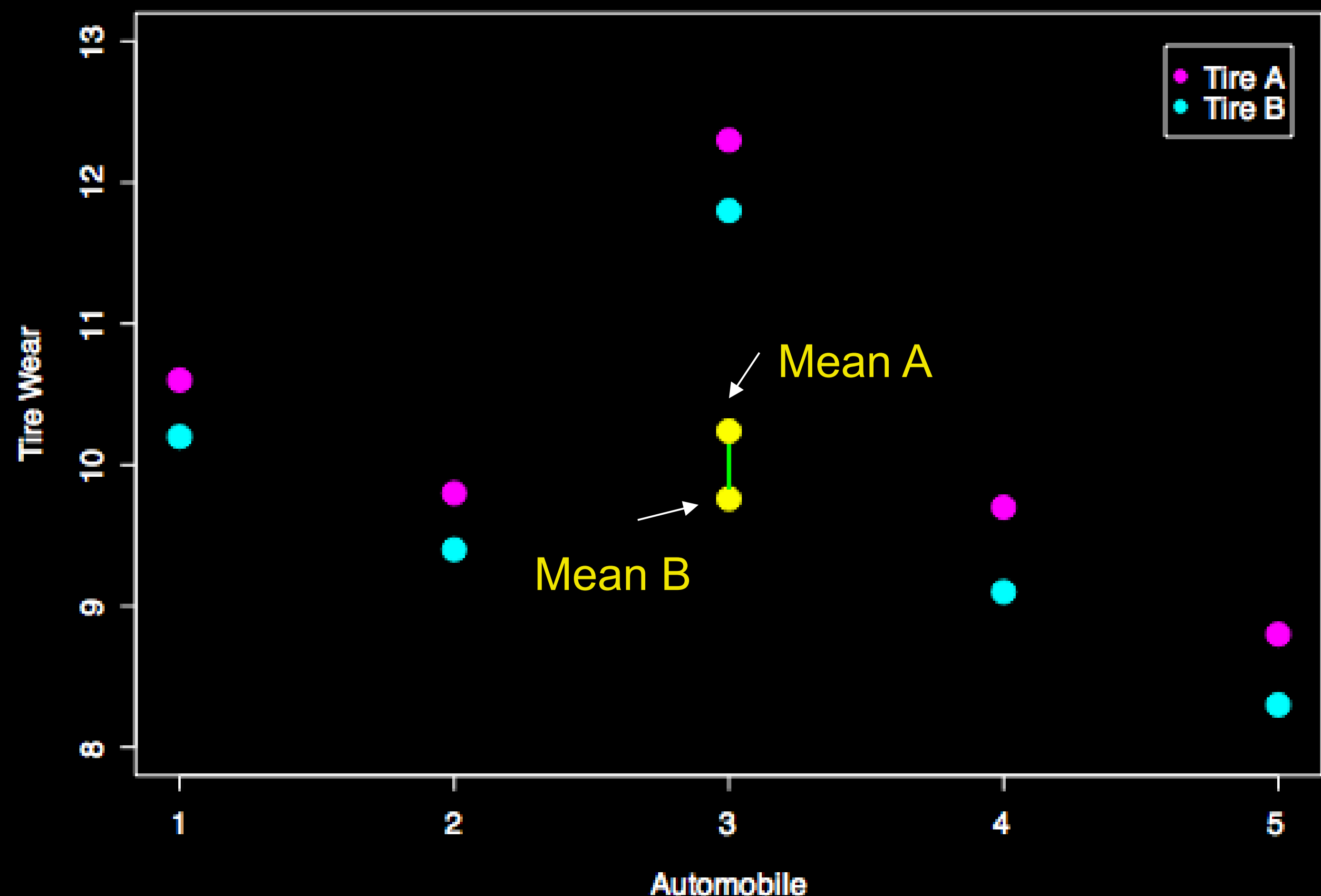
Automobile	Tire A	Tire B
1	10.6	10.2
2	9.8	9.4
3	12.3	11.8
4	9.7	9.1
5	8.8	8.3

- 2-sample T test $p=0.58$
- Paired T test $p<0.001$

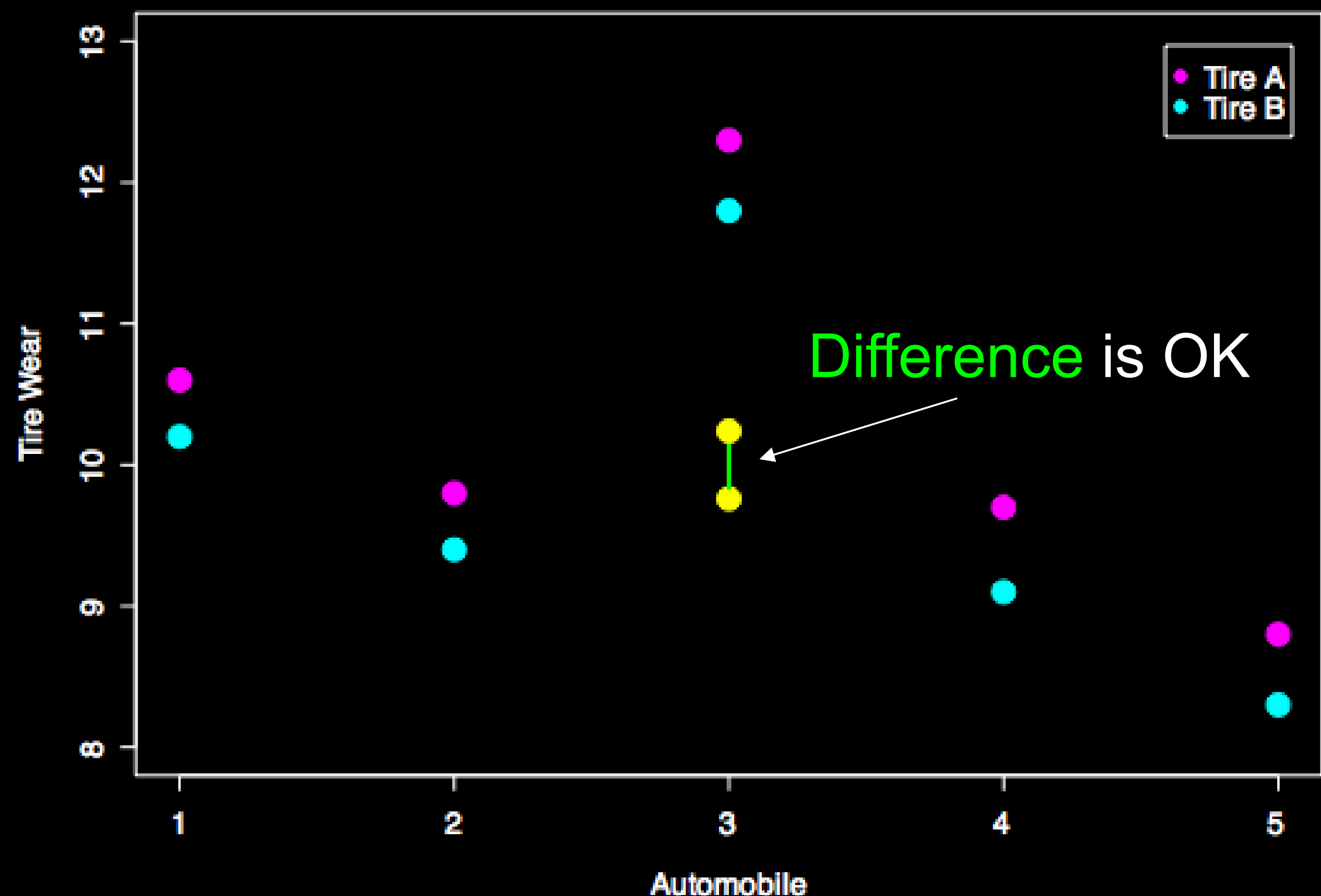
Why so different?



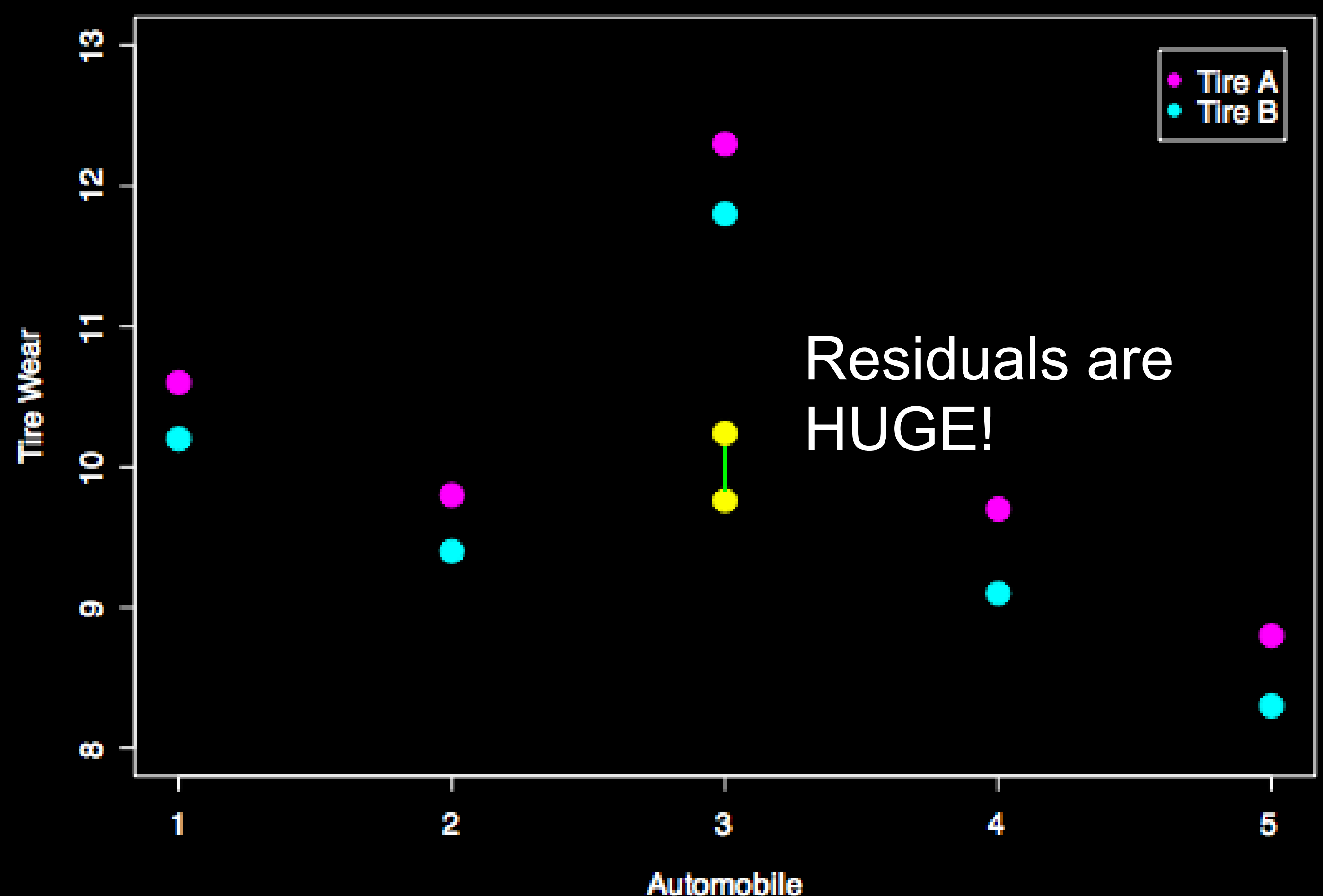
Why so different?



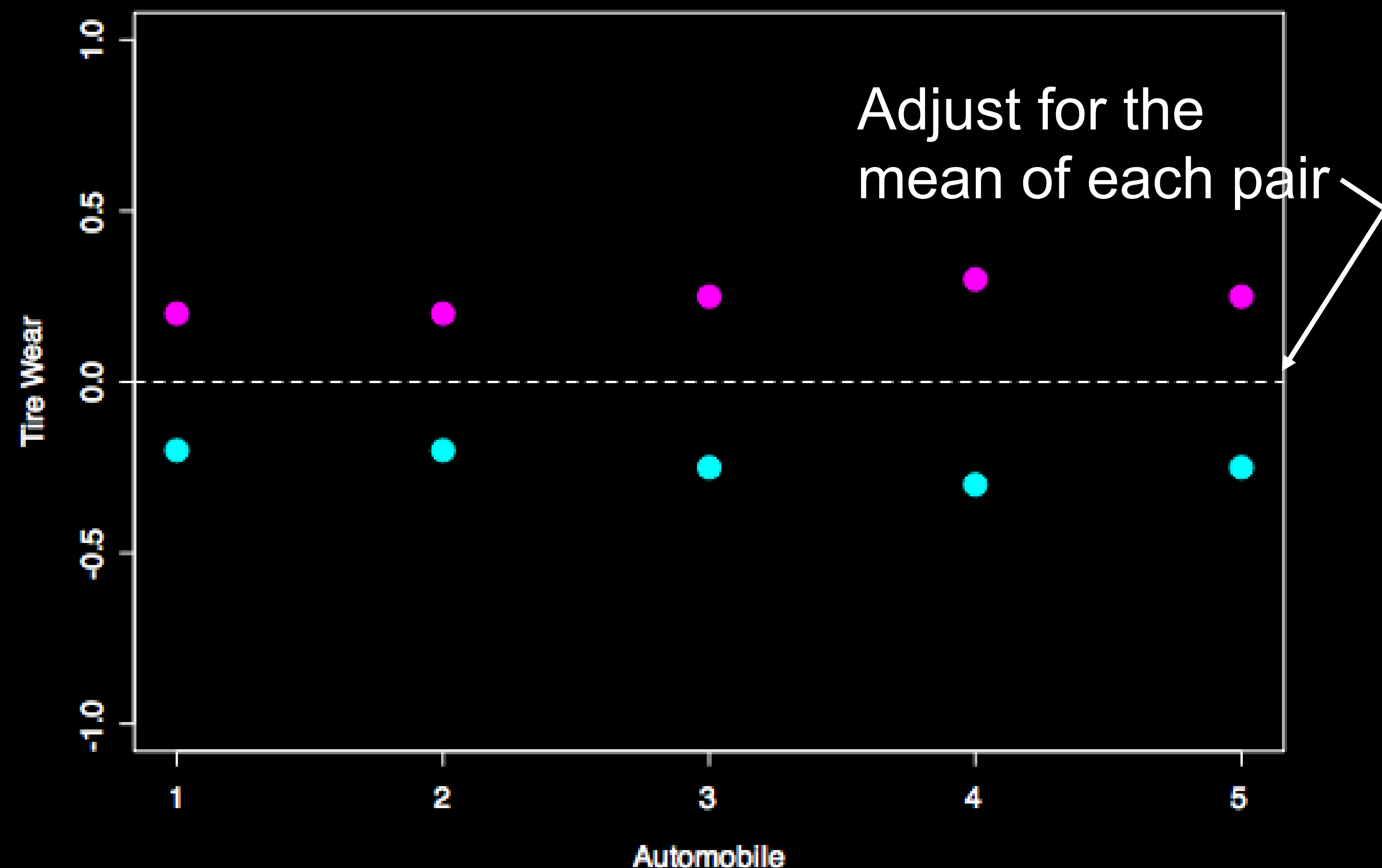
Why so different?



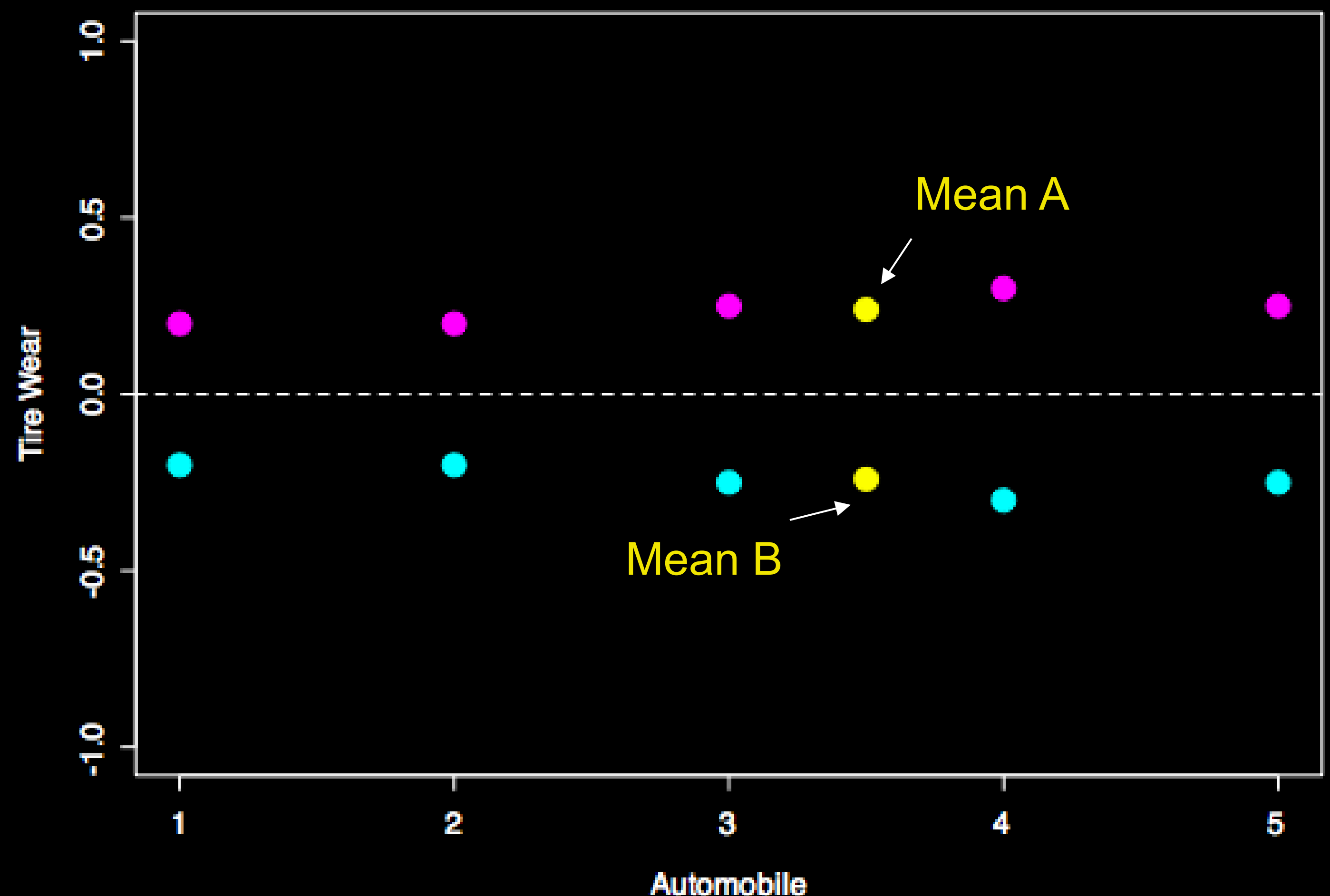
Why so different?



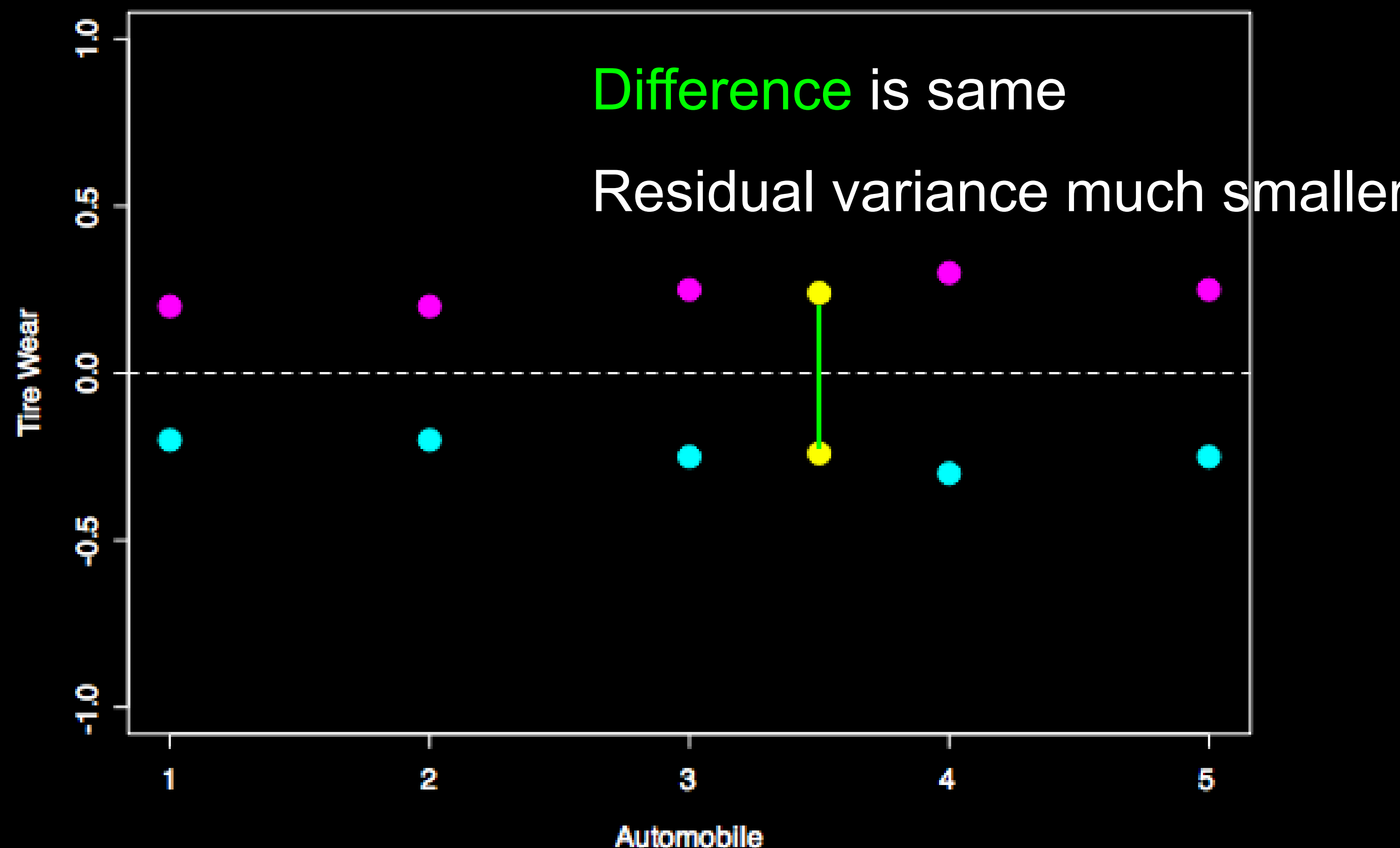
Paired T Test



Paired T Test



Paired T Test



Paired T Test GLM

$$\begin{pmatrix} A_1 \\ B_1 \\ A_2 \\ B_2 \\ A_3 \\ B_3 \\ A_4 \\ B_4 \\ A_5 \\ B_5 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \\ \beta_6 \end{pmatrix}$$

Difference (β_1)

Mean of each pair (β_2–β_6)

$$H_0 : \text{Paired difference} = 0$$
$$H_0 : c\beta = 0, \quad c = [1\ 0\ 0\ 0\ 0\ 0]$$

ANOVA

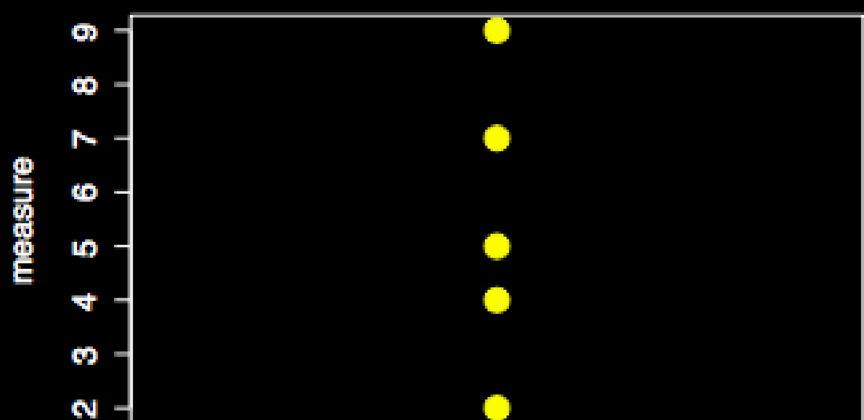
1-way ANOVA

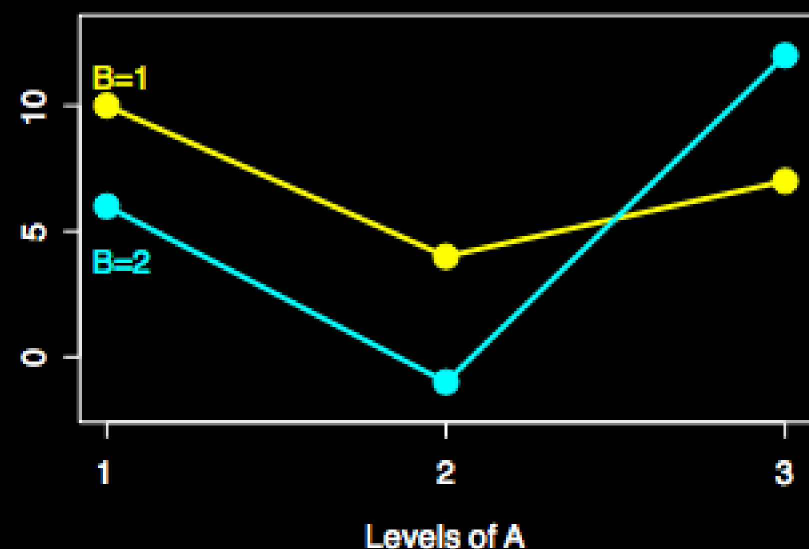
μ_1
μ_2
$\vdots$
μ_N

2-way ANOVA

B

A		
	μ_{11}	μ_{12}
	μ_{21}	μ_{22}
	μ_{31}	μ_{32}





Modeling ANOVA with GLM

- Cell means model
  - 1-way ANOVA $Y_{in} = \mu_i + \epsilon_{in}$
  - 2-way ANOVA $Y_{ijn} = \mu_{ij} + \epsilon_{ijn}$
  - EVs are easy, but contrasts are trickier

Modeling ANOVA with GLM

- Cell means model
  - 1-way ANOVA $Y_{in} = \mu_i + \epsilon_{in}$
  - 2-way ANOVA $Y_{ijn} = \mu_{ij} + \epsilon_{ijn}$
  - EVs are easy, but contrasts are trickier
- Factor effects
  - 1-way $Y_{in} = \mu_. + \alpha_i + \epsilon_{in}$
  - 2-way $Y_{ijn} = \mu_. + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijn}$
  - EVs take more thought, but contrasts are easier
- ANOVA = F tests!

1 Way ANOVA - Cell Means

$$\begin{pmatrix} A_1 \\ A_1 \\ A_2 \\ A_2 \\ A_3 \\ A_3 \\ A_4 \\ A_4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{pmatrix}$$

$$H_0 : G2 - G3 = 0$$

1 Way ANOVA - Cell Means

$$\begin{pmatrix} A_1 \\ A_1 \\ A_2 \\ A_2 \\ A_3 \\ A_3 \\ A_4 \\ A_4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{pmatrix}$$

$$H_0 : G2 - G3 = 0$$

$$H_0 : c\beta = 0 \quad \text{where} \quad c = [0 \;\; 1 \;\; -1 \;\; 0]$$

1 Way ANOVA - Cell Means

$$\begin{pmatrix} A_1 \\ A_1 \\ A_2 \\ A_2 \\ A_3 \\ A_3 \\ A_4 \\ A_4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{pmatrix}$$

$$H_0 : G1 = G2 = G3 = G4 = 0$$

1 Way ANOVA - Cell Means

$$\begin{pmatrix} A_1 \\ A_1 \\ A_2 \\ A_2 \\ A_3 \\ A_3 \\ A_4 \\ A_4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{pmatrix}$$

$$H_0 : G1 = G2 = G3 = G4 = 0$$

$$H_0 : c\beta = 0 \quad \text{where} \quad c = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

1 Way ANOVA - Factor Effects

- In general
  - # of regressors for a factor = # levels-1
  - Factor with 4 levels
    - $X_i = \begin{matrix} 1 & \text{if case from level i} \\ -1 & \text{if case from level 4} \\ 0 & \text{otherwise} \end{matrix}$

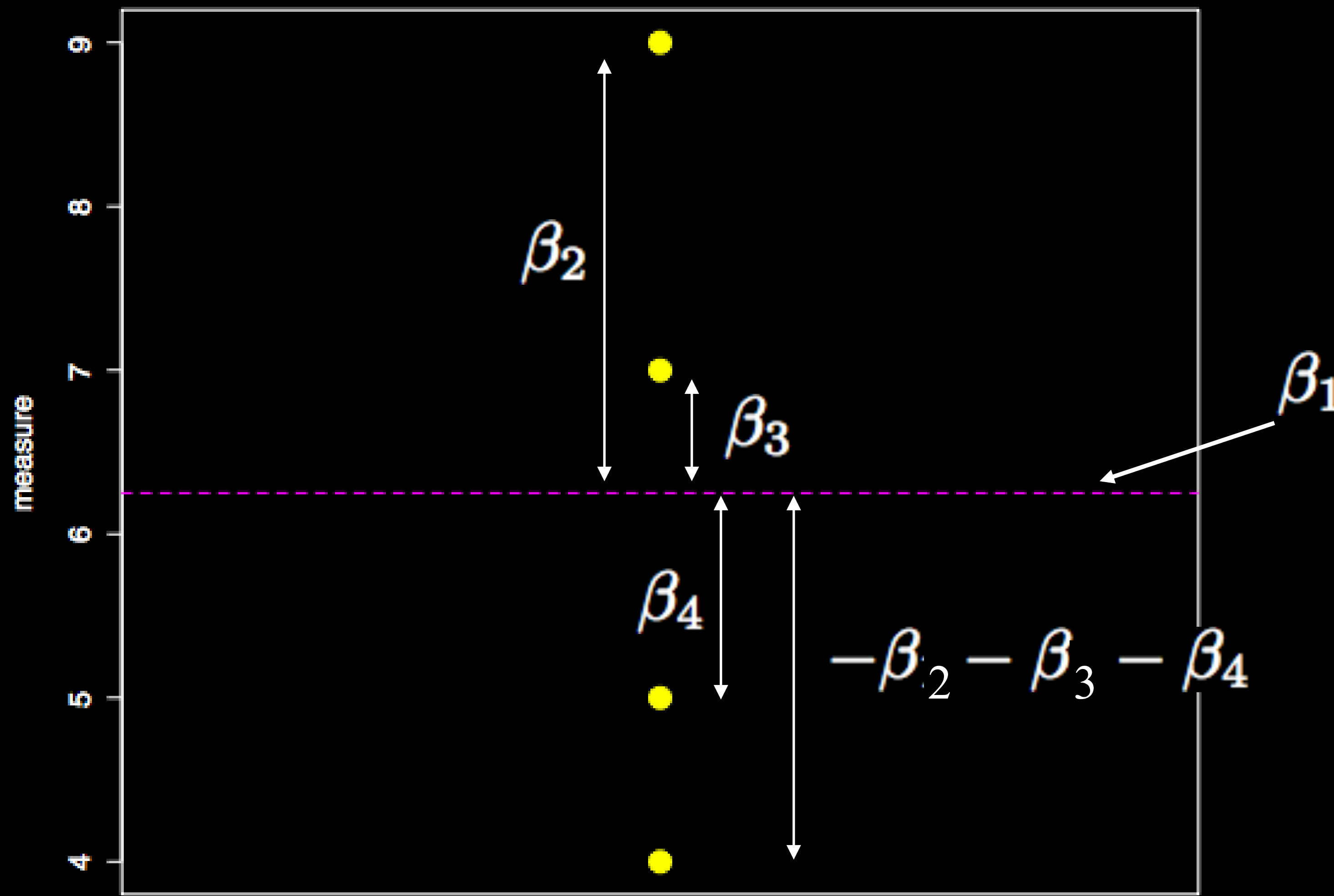
1 Way ANOVA - Factor Effects

$$
\begin{pmatrix} A_1 \\ A_1 \\ A_2 \\ A_2 \\ A_3 \\ A_3 \\ A_4 \\ A_4 \end{pmatrix}
=
\begin{pmatrix}
1 & 1 & 0 & 0 \\
1 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 \\
1 & 0 & 1 & 0 \\
1 & 0 & 0 & 1 \\
1 & 0 & 0 & 1 \\
1 & -1 & -1 & -1 \\
1 & -1 & -1 & -1
\end{pmatrix}
\begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{pmatrix}
$$

mean (β_1)

$$
\begin{aligned}
G1 &= \beta_1 + \beta_2 \\
G2 &= \beta_1 + \beta_3 \\
G3 &= \beta_1 + \beta_4 \\
G4 &= \beta_1 - \beta_2 - \beta_3 - \beta_4
\end{aligned}
$$

1 Way ANOVA - Factor Effects



1 Way ANOVA - Factor Effects

$$\begin{pmatrix} A_1 \\ A_1 \\ A_2 \\ A_2 \\ A_3 \\ A_3 \\ A_4 \\ A_4 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & -1 & -1 & -1 \\ 1 & -1 & -1 & -1 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{pmatrix}$$

$$H_0 : \text{mean of G1} = 0$$

1 Way ANOVA - Factor Effects

$$\begin{pmatrix} A_1 \\ A_1 \\ A_2 \\ A_2 \\ A_3 \\ A_3 \\ A_4 \\ A_4 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & -1 & -1 & -1 \\ 1 & -1 & -1 & -1 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{pmatrix}$$

$$H_0 : \text{mean of G1} = 0$$

$$H_0 : c\beta = 0 \quad \text{where} \quad c = [1 \;\; 1 \;\; 0 \;\; 0]$$

1 Way ANOVA - Factor Effects

$$\begin{pmatrix} A_1 \\ A_1 \\ A_2 \\ A_2 \\ A_3 \\ A_3 \\ A_4 \\ A_4 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & -1 & -1 & -1 \\ 1 & -1 & -1 & -1 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{pmatrix}$$

$$H_0 : G1 - G4 = 0$$

1 Way ANOVA - Factor Effects

$$\begin{pmatrix} A_1 \\ A_1 \\ A_2 \\ A_2 \\ A_3 \\ A_3 \\ A_4 \\ A_4 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & -1 & -1 & -1 \\ 1 & -1 & -1 & -1 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{pmatrix}$$

$$H_0 : G1 - G4 = 0$$

$$c = (1 \;\; 1 \;\; 0 \;\; 0) - (1 \;\; -1 \;\; -1 \;\; -1) = (0 \;\; 2 \;\; 1 \;\; 1)$$

2 Way ANOVA (3x2)

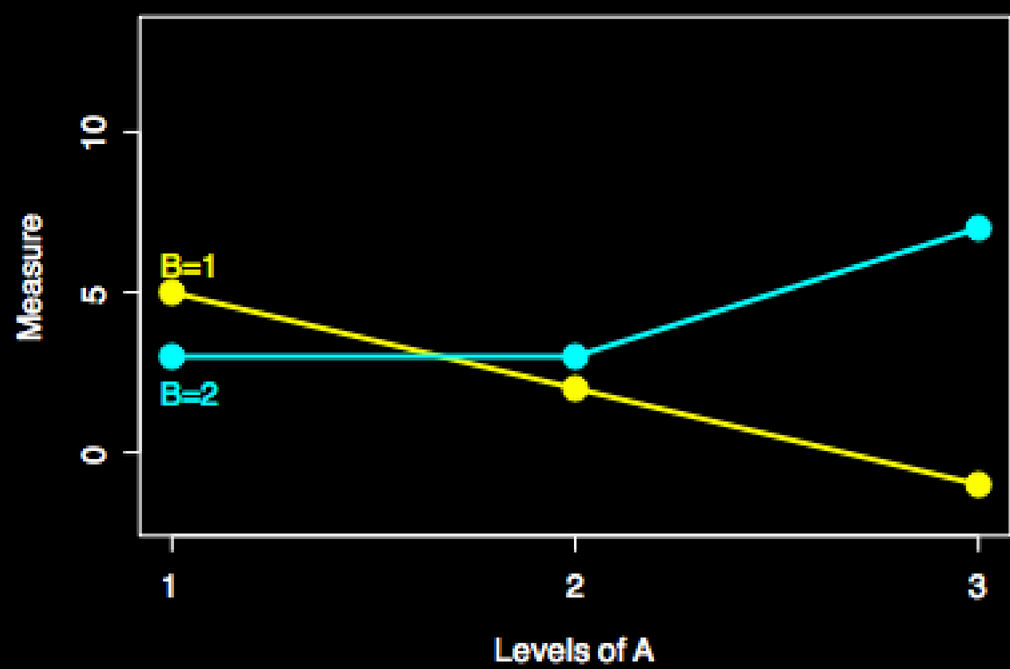
$$\begin{pmatrix} A_1B_1 \\ A_1B_1 \\ A_1B_2 \\ A_1B_2 \\ A_2B_1 \\ A_2B_1 \\ A_2B_2 \\ A_2B_2 \\ A_3B_1 \\ A_3B_1 \\ A_3B_2 \\ A_3B_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \\ \beta_6 \end{pmatrix}$$

$$H_0 : \text{main factor A effect} = 0$$

2 Way ANOVA (3x2)

$$H_0 : \text{main factor A effect} = 0$$

	B1	B2	
A1	5	3	8
A2	2	3	5
A3	-1	7	6
	6	13	19



No effect means the marginals would be the same

Null: A1=A2=A3 equivalently A1-A3=0 and A2-A3=0

2 Way ANOVA (3x2)

$$\begin{pmatrix} A_1B_1 \\ A_1B_1 \\ A_1B_2 \\ A_1B_2 \\ A_2B_1 \\ A_2B_1 \\ A_2B_2 \\ A_2B_2 \\ A_3B_1 \\ A_3B_1 \\ A_3B_2 \\ A_3B_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \\ \beta_6 \end{pmatrix}$$

$$H_0 : \text{main factor A effect} = 0$$

2 Way ANOVA (3x2)

$$\begin{pmatrix} A_1B_1 \\ A_1B_1 \\ A_1B_2 \\ A_1B_2 \\ A_2B_1 \\ A_2B_1 \\ A_2B_2 \\ A_2B_2 \\ A_3B_1 \\ A_3B_1 \\ A_3B_2 \\ A_3B_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \\ \beta_6 \end{pmatrix}$$

$$H_0 : \text{main factor A effect} = 0$$

$$H_0 : c\beta = 0 \quad \text{where} \quad c = \begin{pmatrix} 1 & 1 & 0 & 0 & -1 & -1 \\ 0 & 0 & 1 & 1 & -1 & -1 \end{pmatrix}$$

2 Way ANOVA (3x2)

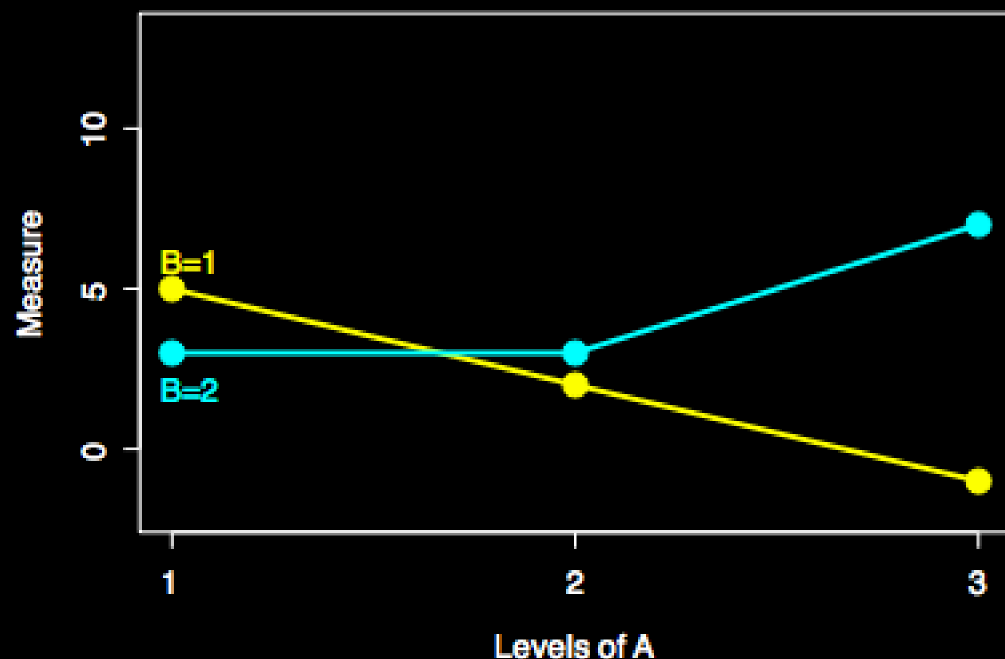
$$\begin{pmatrix} A_1B_1 \\ A_1B_1 \\ A_1B_2 \\ A_1B_2 \\ A_2B_1 \\ A_2B_1 \\ A_2B_2 \\ A_2B_2 \\ A_3B_1 \\ A_3B_1 \\ A_3B_2 \\ A_3B_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \\ \beta_6 \end{pmatrix}$$

$$H_0 : \text{interaction effect} = 0$$

2 Way ANOVA (3x2)

$$H_0 : \text{interaction effect} = 0$$

	B1	B2	
A1	5	3	8
A2	2	3	5
A3	-1	7	6
	6	13	19

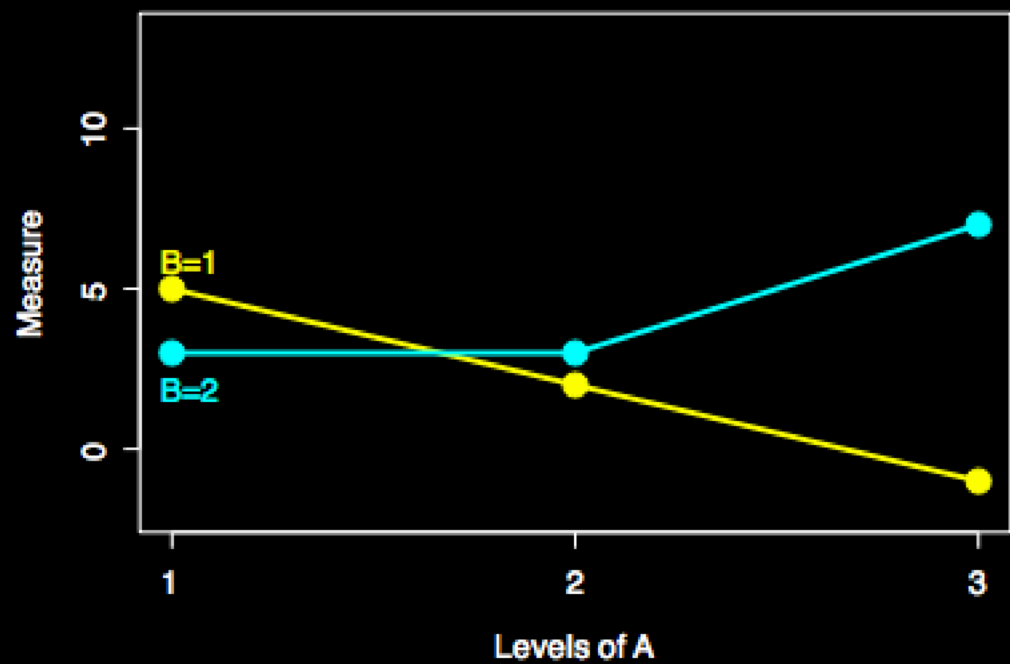


No effect means the lines would be parallel

2 Way ANOVA (3x2)

$$H_0 : \text{interaction effect} = 0$$

	B1	B2	
A1	5	3	8
A2	2	3	5
A3	-1	7	6
	6	13	19



No effect means the lines would be parallel

A1B1-A1B2=A2B1-A2B2=A3B1-A3B2

2 Way ANOVA (3x2)

$$\begin{pmatrix} A_1B_1 \\ A_1B_1 \\ A_1B_2 \\ A_1B_2 \\ A_2B_1 \\ A_2B_1 \\ A_2B_2 \\ A_2B_2 \\ A_3B_1 \\ A_3B_1 \\ A_3B_2 \\ A_3B_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \\ \beta_6 \end{pmatrix}$$

$$H_0 : \text{interaction effect} = 0$$

$$H_0 : c\beta = 0 \quad \text{where} \quad c = \begin{pmatrix} 1 & -1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 & -1 & 1 \end{pmatrix}$$

2 Way ANOVA - Factor Effects

- Recall for factor effects, a factor with n levels has regressors set up like

$$X_i = \begin{matrix} 1 & \text{if case from level i} \\ -1 & \text{if case from level n} \\ 0 & \text{otherwise} \end{matrix}$$

- A has 3 levels, so 2 regressors
- B has 2 levels, so 1 regressor

2 Way ANOVA - Factor Effects

$$
\begin{pmatrix} A_1B_1 \\ A_1B_1 \\ A_1B_2 \\ A_1B_2 \\ A_2B_1 \\ A_2B_1 \\ A_2B_2 \\ A_2B_2 \\ A_3B_1 \\ A_3B_1 \\ A_3B_2 \\ A_3B_2 \end{pmatrix}
=
\begin{pmatrix}
1 & 1 & 0 & 1 & 1 & 0 \\
1 & 1 & 0 & 1 & 1 & 0 \\
1 & 1 & 0 & -1 & -1 & 0 \\
1 & 1 & 0 & -1 & -1 & 0 \\
1 & 0 & 1 & 1 & 0 & 1 \\
1 & 0 & 1 & 1 & 0 & 1 \\
1 & 0 & 1 & -1 & 0 & -1 \\
1 & 0 & 1 & -1 & 0 & -1 \\
1 & -1 & -1 & 1 & -1 & -1 \\
1 & -1 & -1 & 1 & -1 & -1 \\
1 & -1 & -1 & -1 & 1 & 1 \\
1 & -1 & -1 & -1 & 1 & 1
\end{pmatrix}
\begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \\ \beta_6 \end{pmatrix}
$$

A B AB

2 Way ANOVA - Factor Effects

$$\begin{pmatrix} A_1B_1 \\ A_1B_1 \\ A_1B_2 \\ A_1B_2 \\ A_2B_1 \\ A_2B_1 \\ A_2B_2 \\ A_2B_2 \\ A_3B_1 \\ A_3B_1 \\ A_3B_2 \\ A_3B_2 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & -1 & -1 & 0 \\ 1 & 1 & 0 & -1 & -1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & -1 & 0 & -1 \\ 1 & 0 & 1 & -1 & 0 & -1 \\ 1 & -1 & -1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & -1 & -1 \\ 1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & -1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \\ \beta_6 \end{pmatrix}$$

$$H_0 : \text{main factor A effect} = 0$$

2 Way ANOVA - Factor Effects

$$\begin{pmatrix} A_1B_1 \\ A_1B_1 \\ A_1B_2 \\ A_1B_2 \\ A_2B_1 \\ A_2B_1 \\ A_2B_2 \\ A_2B_2 \\ A_3B_1 \\ A_3B_1 \\ A_3B_2 \\ A_3B_2 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & -1 & -1 & 0 \\ 1 & 1 & 0 & -1 & -1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & -1 & 0 & -1 \\ 1 & 0 & 1 & -1 & 0 & -1 \\ 1 & -1 & -1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & -1 & -1 \\ 1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & -1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \\ \beta_6 \end{pmatrix}$$

$$H_0 : \text{main factor A effect} = 0$$

$$H_0 : c\beta = 0 \quad \text{where} \quad c = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

2 Way ANOVA - Factor Effects

$$\begin{pmatrix} A_1B_1 \\ A_1B_1 \\ A_1B_2 \\ A_1B_2 \\ A_2B_1 \\ A_2B_1 \\ A_2B_2 \\ A_2B_2 \\ A_3B_1 \\ A_3B_1 \\ A_3B_2 \\ A_3B_2 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & -1 & -1 & 0 \\ 1 & 1 & 0 & -1 & -1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & -1 & 0 & -1 \\ 1 & 0 & 1 & -1 & 0 & -1 \\ 1 & -1 & -1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & -1 & -1 \\ 1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & -1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \\ \beta_6 \end{pmatrix}$$

$$H_0 : \text{interaction effect} = 0$$

2 Way ANOVA - Factor Effects

$$\begin{pmatrix} A_1B_1 \\ A_1B_1 \\ A_1B_2 \\ A_1B_2 \\ A_2B_1 \\ A_2B_1 \\ A_2B_2 \\ A_2B_2 \\ A_3B_1 \\ A_3B_1 \\ A_3B_2 \\ A_3B_2 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & -1 & -1 & 0 \\ 1 & 1 & 0 & -1 & -1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & -1 & 0 & -1 \\ 1 & 0 & 1 & -1 & 0 & -1 \\ 1 & -1 & -1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & -1 & -1 \\ 1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & -1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \\ \beta_6 \end{pmatrix}$$

$$H_0 : \text{interaction effect} = 0$$

$$H_0 : c\beta = 0 \quad \text{where} \quad c = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

2 Way ANOVA - Factor Effects

$$\begin{pmatrix} A_1B_1 \\ A_1B_1 \\ A_1B_2 \\ A_1B_2 \\ A_2B_1 \\ A_2B_1 \\ A_2B_2 \\ A_2B_2 \\ A_3B_1 \\ A_3B_1 \\ A_3B_2 \\ A_3B_2 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & -1 & -1 & 0 \\ 1 & 1 & 0 & -1 & -1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & -1 & 0 & -1 \\ 1 & 0 & 1 & -1 & 0 & -1 \\ 1 & -1 & -1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & -1 & -1 \\ 1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & -1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \\ \beta_6 \end{pmatrix}$$

$$H_0 : \text{mean cell } A_1B_1 = 0$$

2 Way ANOVA - Factor Effects

$$\begin{pmatrix} A_1B_1 \\ A_1B_1 \\ A_1B_2 \\ A_1B_2 \\ A_2B_1 \\ A_2B_1 \\ A_2B_2 \\ A_2B_2 \\ A_3B_1 \\ A_3B_1 \\ A_3B_2 \\ A_3B_2 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & -1 & -1 & 0 \\ 1 & 1 & 0 & -1 & -1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & -1 & 0 & -1 \\ 1 & 0 & 1 & -1 & 0 & -1 \\ 1 & -1 & -1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & -1 & -1 \\ 1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & -1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \\ \beta_6 \end{pmatrix}$$

$$H_0 : \text{mean cell } A_1B_1 = 0$$

$$H_0 : c\beta = 0 \quad \text{where} \quad c = \begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

For more examples

- The FSL folks have a bunch of great examples
  - http://www.fmrib.ox.ac.uk/fsl/feat5/detail.html
- Check the FSL help list regularly
  - Subscribe at jiscmail
  - Often others have already asked your questions!