Diffusion Tensor Imaging - basic principles

- Diffusion in brain tissues
- Apparent Diffusion Coefficient
- Diffusion Tensor model
- Tensor-derived measures
Diffusion Tensor Imaging (DTI)

Diffusion Tensor Model. In each voxel:

$$S_j = S_0 \exp(-b_j \mathbf{x}_j^T \mathbf{D} \mathbf{x}_j)$$

<table>
<thead>
<tr>
<th>b-value for gradient $j$ (known)</th>
<th>Unit vector representing the direction of gradient $j$ (known)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal measured after applying a Gradient $j$ with direction $\mathbf{x}_j$ and b-value $b_j$ (measured)</td>
<td>3x3 Diffusion Tensor (unknown)</td>
</tr>
</tbody>
</table>
The Elements of the Diffusion Tensor

Tensor is symmetric (6 unknowns)

- **Diagonal Elements** are proportional to the diffusion displacement variances (ADCs) along the three directions of the experiment coordinate system

- **Off-diagonal Elements** are proportional to the correlations (covariances) of displacements along these directions

\[
D = \begin{bmatrix}
D_{xx} & D_{xy} & D_{xz} \\
D_{xy} & D_{yy} & D_{yz} \\
D_{xz} & D_{yz} & D_{zz}
\end{bmatrix}
\]

\[N_3 (0, 2tD)\]
Why do we need a tensor?

\[ \Delta x \quad \Delta y \]
Why do we need a tensor?
Why do we need a tensor?

\[
\begin{bmatrix}
D_x & D_{xy} \\
D_{xy} & D_y
\end{bmatrix}
\]
Once $D$ is estimated, we get ADCs along the scanner’s coordinate system. But we want ADCs along a local coordinate system in each voxel, determined by the anatomy.
The Diffusion Tensor Eigenspectrum

Once $D$ is estimated, we get ADCs along the scanner’s coordinate system. But we want ADCs along a local coordinate system in each voxel, determined by the anatomy.

Diagonalize the estimated tensor in each voxel

$$D = \begin{bmatrix}
D_{xx} & D_{xy} & D_{xz} \\
D_{xy} & D_{yy} & D_{yz} \\
D_{xz} & D_{yz} & D_{zz}
\end{bmatrix}$$

$D = [v_1|v_2|v_3]^T \begin{bmatrix}
\lambda_1 & 0 & 0 \\
0 & \lambda_2 & 0 \\
0 & 0 & \lambda_3
\end{bmatrix} [v_1|v_2|v_3]$
The Diffusion Tensor Ellipsoid

**Isotropic voxel**

\[ \lambda_1 \approx \lambda_2 \approx \lambda_3 \]

**Anisotropic voxel**

\[ \lambda_1 \gg \lambda_2, \lambda_3 \]

\[ \mathbf{v}_1 \sqrt{2 \tau \lambda_1} \]

\[ \mathbf{v}_2 \sqrt{2 \tau \lambda_2} \]

\[ \mathbf{v}_3 \sqrt{2 \tau \lambda_3} \]
The Diffusion Tensor Ellipsoid

- CSF
- Grey matter
- White matter
Quantitative Diffusion Maps

Fractional Anisotropy (FA) \sim \text{Eigenvalues Variance (normalised)}

Mean Diffusivity (MD) = \text{Eigenvalues Mean}

\[ FA = \sqrt{\frac{3 \sum_{i=1}^{3} (\lambda_i - \bar{\lambda})^2}{2 \sum_{i=1}^{3} \lambda_i^2}}, \quad FA \text{ in } [0,1] \]

\[ MD = \frac{D_{xx} + D_{yy} + D_{zz}}{3} = \frac{\lambda_1 + \lambda_2 + \lambda_3}{3} \]
Quantitative Diffusion Maps
Quantitative Diffusion Maps

FA

MD

Longitudinal/axial/parallel ADC \((\lambda_1)\)

Transverse/radial/perpendicular ADC \((\lambda_2 + \lambda_3)/2\)
Quantitative Diffusion Maps

FA decrease/ MD increase has been associated in many studies with tissue breakdown (loss of structure).


Fractional Anisotropy changes in MS normal appearing white matter
Quantitative Diffusion Maps

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Fractional Anisotropy changes in MS normal appearing white matter
Quantitative Diffusion Maps

Different scenarios can have same effect on FA, MD

- Swelling
- Higher Density
- Myelin Loss
- Cell Death
Tensor and FA in Crossing Regions

- In voxels containing two crossing bundles, FA is low and the tensor ellipsoid is pancake-shaped (oblate, planar tensor).
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Diffusion Tensor Ellipsoids
Estimates of Principal Fibre Orientation in WM

**Assumption!!**

Direction of maximum *diffusivity* in voxels with anisotropic profile is an estimate of the major fibre orientation.
Estimates of Principal Fibre Orientation in WM
Directional contrast in DTI